# Beyond the Vehicle Routing Problem: Design of Temporal Networks for Demand-Responsive Transport

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Abstract: Conventional public transportation (CPT) is composed of fixed routes and fixed timetables, usually determined via long-term planning, based on nominal demand. However, during operations, demand may greatly deviate from the nominal one, causing a mismatch between demand and supply, leading to an inefficient service. On the other hand, flexible mobility services, such as Demand-Responsive Transport (DRT), adapt bus routes to the actual user demand. However, routes are calculated by solving a Vehicle Routing Problems (VRPs), which are not as effective as CPT in terms of demand consolidation, resulting in cost inefficiency. While in CPT, consolidation is obtained by forcing users to adapt to CPT by lines, VRP adapts instead to bus routes to user demand. This work introduces an alternative approach to DRT operations: different from VRP, we design a structured network describing bus routes, allowing for complex user trips, including transfers and walking legs. This enables greater consolidation and efficiency. While network design problems are limited to static networks, we propose here an original formulation to design temporal networks, which allows structured bus routes to adapt to the observed demand. We provide a proof-of-concept of the proposed approach, and show in small-scale numerical experiments that it reduces operator cost, without excessively penalizing users, compared to the classic VRP-based solution (Code available at https://github.com/XiaoyiWu21/TN4DRT).

## 1 INTRODUCTION

Public transportation (PT) plays a vital role in urban mobility systems, offering affordable prices, reducing accident-related fatalities, and mitigating externalities associated with private vehicle use (Mackett, 2022). Conventional public transit (CPT) systems typically operate on fixed routes and schedules, a design that provides high reliability without disruptions. However, the fixed design has limited adaptability when the travel demand significantly deviates from average scenarios, such as sparsely populated suburban regions (O'Shaughnessy et al., 2011; Sörensen et al., 2021) or during off-/peak hours. These limitations can constrain access to mobility and exacerbate transport inequities (Giuliano, 2005).

With the advancement in real-time location and digital application technologies on phones, Mobility-on-Demand (MoD) services have emerged as a flexible alternative to CPT without requiring high infrastructure expenses. By coordinating vehicles dynamically in response to user requests, MoD provides more timely and adaptive services for travelers. Nevertheless, this flexibility could introduce high uncertainty in pickup and dropoff times and location and thus reduce travelers' willingness to pay (Li et al., 2010). Moreover, in the absence of effective demand consolidation, MoD can increase Vehicle-Kilometers Traveled (VKT) and cannibalize PT ridership by attracting users who have a higher willingness to pay for the door-to-door services (Oh et al., 2020, 2021; Gurumurthy and Kockelman, 2022; Araldo et al., 2019).

These trade-offs have motivated growing interest in demand-responsive transit (DRT) (Vansteenwegen et al., 2022; Melis and Sörensen, 2022b,a), which seeks to bridge the gap between CPT and MoD by designing flexible and efficient transit systems that adapt to spatial and temporal variations.

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However, most existing DRT studies determine routes by solving Vehicle Routing Problems (VRPs). While providing flexible mobility for non-nominal demand areas, VRP-based approaches focus on door-to-door service and are often less cost-efficient than CPT due to limited demand consolidation. To address this gap, this study introduces a new formulation for DRT optimization based on temporal network design, which jointly optimizes vehicle routing and timetables while enhancing user aggregation.

In addition, previous work has shown that incorporating short walking segments when determining pickup and dropoff locations can reduce detours and improve overall system efficiency (Fielbaum, 2022; Fielbaum et al., 2021; Araldo et al., 2019). Evidence from stated-preference experiments further indicates that users are more willing to walk when the walking segment is short, well-informed, and directionally convenient (Pellegrini and Fielbaum, 2025). Accordingly, our model endogenizes first- and last-mile walking, as well as the option of full-walk trips, within the optimization framework.

Moreover, Masson et al. (2014) shows that the enabling transfer in Dial-a-Ride Problems (DARP) can significantly reduce vehicle travel distances. Building on this insight, our formulation explicitly integrates transfers between vehicles, with formal constraints that ensure service feasibility for protected users.

In summary, the contributions of this study are threefold. First, we propose a novel temporal network formulation for DRT service optimization, which enhances system efficiency and user experience by consolidating demand and explicitly modeling transfers and walking. Second, we develop a proof-of-concept implementation to demonstrate the feasibility of our proposed approach. Third, through small-scale numerical experiments, we show that the proposed method reduces operator costs while maintaining quality of service for users, outperforming VRP-based solutions.

In the following sections, we first review related work in section 2 and then formulate the problem in section 3. Next, we describe the numerical experiment and corresponding results in section 4. Finally, we summarize the conclusions and discuss future potential work in section 5.

# 2 LITERATURE REVIEW

This work lies at the intersection of three major research domains: *Vehicle Routing Problems (VRP)*, *Conventional Public Transit (CPT)* design, and *Demand-Responsive Transit (DRT)* systems. Each of these domains approaches the challenge of providing efficient and effective mobility services for the public from a distinct perspective.

## 2.1 Vehicle Routing Problems

The Vehicle Routing Problem (VRP) is a classical combinatorial optimization problem with widespread applications in transportation and logistics. A recent comprehensive review can be found in Zhang et al. (2022). However, due to its NP-hard nature, VRP remains a challenging problem to solve efficiently, especially at scale. With recent advances in the field of Artificial Intelligence (AI), many AI-based approaches have been proposed that offer strong generalization capabilities—meaning the same model can adapt to various VRP variants without the need to redesign the algorithm. Nazari et al. (2018) formulated VRP as a Markov Decision Chain and used reinforcement learning to solve it by deciding the node for the vehicle to visit in sequence.

There is a branch of VRP similar to our problem: The pickup and delivery problem with time windows (PDPTW) (Dumas et al., 1991). In these problems, the VRP solvers need to find the shortest path for the vehicle to pick up and deliver passengers within the time constraints. Kim et al. (2023) extends this problem into rolling horizon optimization for para-transit services through a temporal decomposition.

However, classical VRP formulations are inherently *path-based* and focus on door-to-door operations, which limits their ability to capture transfers or first-/last- mile connectivity across a broader multi-modal transport system.

## 2.2 Conventional Public Transit Design

Efforts to design and optimize PT networks typically focus on the decision-making process across three hierarchical levels: strategic, tactical, and operational (Gkiotsalitis, 2020).

At the strategic level, optimization involves route design and line planning. Borndörfer et al. (2007) proposed an extended multi-commodity flow formulation for the line planning problems (LPP) that minimizes a weighted combination of total passenger travel times and operating costs. While in their work, passengers can transfer between lines within capacity constraints; these transfers are not explicitly modeled (Gkiotsalitis, 2020). Moreover, the model employs aggregate demand flows rather than individual passenger requests, limiting its adaptability to temporal or spatial demand fluctuations.

The tactical level involves adjusting timetabling and service frequencies to balance passenger waiting time and operational costs. Space–time graph models combined with integer linear programming (ILP) are widely used for timetabling. In these models, nodes represent station–time pairs, and edges represent vehicle movements or dwell times. The optimization model determines dwell durations and departure times. However, many such models do not explicitly include transfer edges, limiting their ability to capture passenger transfer dynamics.

Finally, the operational level addresses vehicle and driver scheduling, which define how the PT system is operated on a daily basis.

This traditional sequential optimization approach, though intuitive, is essentially greedy and cannot guarantee global optimality (Schmidt and Schöbel, 2024). Fully integrated formulations are often computationally intractable, motivating research into combined optimization of the LPP and timetabling to achieve practical efficiency (Durán-Micco and Vansteenwegen, 2022).

In our formulation, we extend the space–time graph models to an expanded space–time graph, which allows full flexibility in route and timetable construction. This integrated design jointly optimizes line routes and timetabling, and passenger boarding/alighting stops to minimize detours and enhance synchronization between transit services and passenger demand.

Notably, a closely related research area is the Transit Network Design Problem (TNDP) (Durán-Micco and Vansteenwegen, 2022), which also focuses on defining the lines of a transit network. The terms TNDP and LPP are often used interchangeably, although, as noted by Schmidt and Schöbel (2024), TNDP is more common in transportation engineering, while LPP is prevalent in operations research. When vehicle sizes and frequencies are jointly optimized, the problem becomes the Transit Network Design and Frequency Setting Problem (TNDFSP). As highlighted by Gkiotsalitis (2020), the TNDFSP typically assumes that the physical infrastructure (i.e., stops and road segments) is given, and seeks to determine the number of lines, their routes, and corresponding frequencies.

Both TNDFSP and the integrated optimization of LPP and timetabling are NP-hard problems, and hence, many studies adopt heuristic or metaheuristic approaches to achieve computationally feasible solutions. However, since our work focuses on providing a proof-of-concept of the proposed formulation based on exact algorithms, we do not elaborate further on these heuristic methods for scalability considerations (Montenegro et al., 2021; Blanco et al., 2020).

Balakrishnan et al. (2017) provide an alternative perspective on transportation network design based on a multicommodity network formulation. They introduce the Network Design with Service Requirements (NDSR) problem, which requires not only selecting the arcs to include in the network from the operator side, but also routing each commodity to minimize the total fixed and routing costs subject to commodity-specific service constraints. However, their approach primarily focuses on end-to-end services and does not account for transfers or walking.

## 2.3 Demand-Responsive Transport

DRT has been studied for more than four decades as a flexible mobility alternative between CPT and MoD systems. A comprehensive overview of this research can be found in Vansteenwegen et al. (2022). DRT can have multiple objectives such as reducing operational costs, providing feeder services to other transport modes, reducing environmental impacts, serving populations with limited mobility options, complementing gaps in existing public transport networks, improving accessibility in rural areas, enhancing connectivity between rural, peri-urban, and urban regions, and supporting touristic mobility (Dauer et al., 2024).

Vansteenwegen et al. (2022) present a unified classification framework for DRT systems based on four key characteristics: dynamism, trip structure, flexibility, and objective.

Within this broad context, one specific branch of DRT research, the On-Demand Bus Routing Problem (ODBRP) proposed by Melis and Sörensen (2022b), is particularly relevant to our study. This branch of work focuses on many-to-many, fully flexible DRT systems. Melis and Sörensen (2022b) first proposed the static version of this problem based on the Dial-a-Ride Problems, the School Bus Routing Problems, and PDPTW. Then they studied a dynamic version of this problem based on a bi-level optimization architecture in Melis and Sörensen (2022a). Their results show that assigning users to specific bus stations for boarding and alighting can reduce route detours and operational costs. Lian et al. (2023) extended this framework by incorporating dynamic control of bus departure and arrival times under time-dependent travel conditions.

Nevertheless, most existing DRT formulations inherit VRP's operational structure, optimizing routes for vehicles with limited demand consolidation. As a result, they rarely account for coordinated transfers, first-/last- mile walking, or user movement synchronization.

Based on the above research gap, our work introduces a novel formulation of DRT as a *temporal network design problem* rather than a conventional routing problem. This perspective enables the joint optimization of routes, timetables, and passenger movements within a unified, graph-based framework.

# 3 METHODOLOGY

In this work, we formulate the DRT system using a time-expanded graph that captures both vehicle and passenger movement. This graph forms the foundation of our optimization model and allows us to explicitly encode temporal feasibility, transfers, waiting times, and passenger movement.

## 3.1 Preliminaries

Let $\mathcal{V} = \{0, 1, \ldots, v\}$ denote the set of candidate bus stops, $\mathcal{L} = \{0, 1, \ldots, L\}$ denote the set of vehicle lines, and $\mathcal{T} = \{0, \Delta t, \ldots, T\}$ denote the discretized time intervals.

We denote the set of user requests as $\mathcal{K} = \{0, \ldots, K\}$. Each request $k \in \mathcal{K}$ is represented by a tuple $k = (\mathrm{IDT}^k,\ o^k,\ d^k,\ m^k)$, where $\mathrm{IDT}^k$ is the user's ideal departure time, and $o^k,\ d^k \in \mathbb{R}^2$ are the origin and destination locations, each determined by latitude and longitude. $m^k \in M$ indicates the user's travel preference, which is described in detail in Section 3.1.2.

Based on the ideal departure time, the ideal travel time (ITT) of user $k$ is defined as $\mathrm{ITT}^k = tt^b_{o^k,d^k}$, where $tt^b_{o^k,d^k}$ is the bus travel time between the origin-destination (OD) pair based on Euclidean distance. Consequently, the ideal arrival time (IAT) is given by $\mathrm{IAT}^k = \mathrm{IDT}^k + \mathrm{ITT}^k$.

Applying the method used in Kim et al. (2023), each user's latest pickup time (LPUT) is defined by $\mathrm{LPUT}^k = \mathrm{IDT}^k + \mathrm{WAIT}_{\max}$, where $\mathrm{WAIT}_{\max}$ is the maximum waiting time. Similarly, the latest arrival time (LAT) is defined by $\mathrm{LAT}^k = \mathrm{IAT}^k + \mathrm{DELAY}_{\max}$, where $\mathrm{DELAY}_{\max}$ is the maximum delay.

Subset $\mathcal{K}' \subseteq \mathcal{K}$ contains *protected trips*: These are fictitious requests that have not been issued by any user but whose requirements are to be satisfied anyway when building the network. The set $\mathcal{K}'$ prevents *feedback loop bias* (Pagan et al., 2023), which is a typical issue of data-driven systems (such as DRT or ride sharing), which naturally configure themselves to favor the "prevalent demand" and can automatically (and inadvertently) disfavor the demand that deviates from the prevalent patterns. In the case of mobility, a fully data-driven system may inadvertently exclude (or serve more poorly) those users who are in less dense or disadvantaged areas, hampering their right to mobility (Jiao and Wang, 2021; Barajas and Brown, 2021; Guo, 2024). Imposing $\mathcal{K}'$ represents a "guard" against uncontrolled drifts of demand-driven transport systems toward inequitable behavior. As a starting point, we set $\mathcal{K}' = 0$. An important part of future work involves deciding how to appropriately define $\mathcal{K}'$, namely the ODs that enter and the time distance between them.

The parameters used in the numerical setting are summarized in Table 1.

Table 1: Parameter settings.

| Parameter | Value | Description |
|---|---|---|
| $s_c$ | 14 | Car speed (m/s). |
| $s_b$ | 14 | Bus speed (m/s). |
| $s_w$ | 1.8 | Walking speed (m/s). |
| $\mathrm{FW}_{\max}$ | 30 | Maximum walking time directly from the traveler's OD pair (min). |
| $\mathrm{FW}od^k$ | – | Travel time of user $k$'s OD pair by walk (min). |
| $\mathrm{HOLD}_{\max}$ | 30 | Maximum time on waiting edge (min). |
| $\mathrm{TRAVERSE}_{\max}$ | 30 | Maximum time on traverse edge (min). |
| $\mathrm{TRANSFER}_{\max}$ | 30 | Maximum time on transfer edge (min). |
| $\mathrm{WAIT}_{\max}$ | 20 | Threshold for latest pickup time (min). |
| $\mathrm{DELAY}_{\max}$ | 30 | Threshold for latest arrival time (min). |
| $\alpha_m$ | 0.33 | Weight between operator cost and user cost. |
| $\lvert\mathcal{L}\rvert$ | 2 | Number of vehicle in the DRT system. |
| $\lvert\mathcal{K}\rvert$ | 8–10 | Number of trips for each instance. |
| $\Delta t$ | 1 | Time discretization size (min). |

### 3.1.1 Graph Construction

Let $G = (\mathcal{N}, \mathcal{E})$ denote the time-expanded graph. The node set $\mathcal{N}$ represents all feasible combinations of lines (aka vehicles), stops, and time instances, while the edge set $\mathcal{E}$ represents all possible feasible connections between nodes. Constraints could be imposed on graph construction to reduce computational complexity.

A **node** $n = (l, v, t) \in \mathcal{N}$ represents a *stop-time* tuple (in GTFS terminology): stop $v$ served by line $l$ at time $t$. The node set is partitioned as $\mathcal{N} = \mathcal{N}^{\text{in}} \cup \{src, snk\}$, where *src* and *snk* denote super-source and super-sink nodes, respectively. Internal nodes other than the depot are constructed in an "exhaustive" way: $\mathcal{N}^{\text{in}} = \mathcal{L} \times \mathcal{V} \times \mathcal{T}$.

An **edge** $e = (n, n') = ((l, v, t),\ (l', v', t')) \in \mathcal{E}$ represents a feasible connection between two stop-time nodes. The edge set is partitioned into five mutually disjoint categories, $\mathcal{E} = \mathcal{E}^{\text{hld}} \cup \mathcal{E}^{\text{trv}} \cup \mathcal{E}^{\text{tsf}} \cup \mathcal{E}^{\text{src}} \cup \mathcal{E}^{\text{snk}}$, corresponding to holding, traverse, transfer, source, and sink edges, respectively.

First, when two nodes on the edge belong to the same line ($l = l'$), the connection is either a holding or traverse edge. If they correspond to the same stop at different time instances, the edge belongs to **holding edge** $\mathcal{E}^{\text{hld}}$, allowing users to wait for service at stop $v$ from time $t$ to $t'$ on line $l$. The waiting time is bounded by $t \leq t' \leq t + \text{HOLD}_{\max}$, where $\text{HOLD}_{\max}$ is the maximum allowed waiting time at stop $v$.

If $v \neq v'$, the vehicle travels from stop $v$ to stop $v'$ along a **traverse edge** $\mathcal{E}^{\text{trv}}$. The time difference on a traverse edge should be larger than the bus travel time $tt^{b}_{v,v'}$ and less than the maximum travel time threshold $\text{TRAVERSE}_{\max}$: $t + tt^{b}_{v,v'} \leq t' \leq t + \text{TRAVERSE}_{\max}$.

If the two nodes share the same physical stop but belong to different lines, then the corresponding edge is a **transfer edge** $\mathcal{E}^{\text{tsf}}$. This allows users to change lines at stop $v$ within the maximum transfer waiting time $\text{TRANSFER}_{\max}$: $t \leq t' \leq t + \text{TRANSFER}_{\max}$.

Each line has a stop $v$ connected to the super-source node through **source edges**, $\mathcal{E}^{\text{src}} = \{(src, (l, v, t)) \mid l \in \mathcal{L}, v \in \mathcal{V}, t > tt^{b}_{\text{src},v}\}$, where the service at stop $v$ cannot begin earlier than the DRT bus travel time from the super-source node.

Similarly, each line has a stop connected to the super-sink node through **sink edges**, $\mathcal{E}^{\text{snk}} = \{((l, v, t), snk) \mid l \in \mathcal{L}, v \in \mathcal{V}, t \in \mathcal{T}\}$. For notational convenience, for each node $(l, v, t) \in \mathcal{N}$, let $\mathcal{E}^{-}(l, v, t) = \{e \in \mathcal{E} \mid e \text{ leaves } (l, v, t)\}$ denote its outgoing edge sets and $\mathcal{E}^{+}(l, v, t) = \{e \in \mathcal{E} \mid e \text{ enters } (l, v, t)\}$ denote its incoming edge sets.

For each line $l$, we define a non-transfer edge set associated with that line as $\mathcal{E}^{\text{line}}_{l} := \mathcal{E}^{\text{hld}}_{l} \cup \mathcal{E}^{\text{trv}}_{l} \cup \mathcal{E}^{\text{src}}_{l} \cup \mathcal{E}^{\text{snk}}_{l}$.

For any node $n = (l, v, t)$, we define the subsets of edges incident to $n$ on line $l$ by

$$\mathcal{E}^{+}_{l}(n) := \mathcal{E}^{+}_{l}(l, v, t) = \mathcal{E}^{+}(l, v, t) \cap \mathcal{E}^{\text{line}}_{l},$$
$$\mathcal{E}^{-}_{l}(n) := \mathcal{E}^{-}_{l}(l, v, t) = \mathcal{E}^{-}(l, v, t) \cap \mathcal{E}^{\text{line}}_{l},$$

and the complete set of incident edges on line $l$ is $\mathcal{E}_{l}(n) := \mathcal{E}^{+}_{l}(n) \cup \mathcal{E}^{-}_{l}(n)$.

### 3.1.2 Cost Calculation

The cost structure in our approach consists of two components: *operator cost* and *user cost*.

The *operator cost* corresponds to the cost of activating an edge and is defined as the vehicle travel time (VTT) on that edge. Specifically, for an edge $e$, the operator cost is given by $F_e = d_e / s_b$, where $d_e$ denotes the Euclidean distance of edge $e$, and $s_b$ is the constant vehicle speed.

The *user cost* $c^{k}_{m}$ represents a cost proxy for user $k$ under objective $m \in \mathcal{M}$. We consider six objectives: $\mathcal{M} = \{m_{\text{vtt}}, m_{\text{ivt}}, m_{\text{wait}}, m_{\text{walk}}, m_{\text{tsf}}, m_{\text{com}}\}$.

Objective $m_{\text{vtt}}$ is operator-based and serves as a baseline that does not explicitly account for user cost. The remaining five objectives capture distinct aspects of service quality, namely in-vehicle time, waiting time, walking time, transfer time, and a composite measure of these components, respectively.

For users who complete their entire trip on foot, when the objective accounts for walking time (i.e., $m_{\text{walk}}$ or $m_{\text{com}}$), their user cost equals the walking travel time between their OD pair, denoted by $t^{w}_{o^k, d^k}$. Under the remaining objectives, their user cost is set to zero.

For users using the DRT system, the user cost consists of three components: *edge cost*, *ingress cost*, and *egress cost*.

The *edge cost* $c^{k}_{m,e}$ represents the cost incurred by user $k$ when traversing edge $e$. Although this cost may generally vary across users, we assume homogeneous values across all users, i.e., $c^{k}_{m,e} = c^{k'}_{m,e}$ for all $k, k' \in \mathcal{K}$.

The *ingress cost* $c^{k,\text{in}}_{m,n}$ is associated with the initial leg of the trip and includes walking from the origin $o^k$ to the boarding timestop $v$, as well as waiting until the boarding time $t$.

The *egress cost* $c^{k,\text{out}}_{m,n}$ corresponds to the final leg of the trip and includes walking from the alighting stop $v$ to the destination $d^k$.

In summary, for users who do not complete their trip entirely on foot, the total user cost is given by

$$c^{k}_{m} = \underbrace{\sum_{e \in \mathcal{E}} c^{k}_{m,e} x^{k}_{e}}_{\text{Edge cost}} + \underbrace{\sum_{n \in \mathcal{N}} c^{k,\text{in}}_{m,n} b^{k}_{n}}_{\text{Ingress cost}} + \underbrace{\sum_{n \in \mathcal{N}} c^{k,\text{out}}_{m,n} h^{k}_{n}}_{\text{Egress cost}}, \tag{1}$$

where $x^{k}_{e}$ indicates whether user $k$ traverses edge $e$, and $b^{k}_{n}$ and $h^{k}_{n}$ indicate whether user $k$ boards or alights at time-stop $n$, respectively.

The calculation of each component depends on the selected objective. A summary of the specific cost formulations is provided in Table 2.

Following Balakrishnan et al. (2017), service constraints can be imposed to ensure minimum service quality. A path for user $k$ is considered *feasible* if the corresponding user cost does not exceed a predefined budget, i.e., $c^{k}_{m} \leq W_m$. If a particular service requirement is not enforced, the corresponding budget $W_m$ is set to $+\infty$.

## 3.2 Optimization

Building a DRT network translates into deciding which edges to activate and also to ensure that, for each request $k \in \mathcal{K}$, there exists at least one feasible path. We call it *guaranteed* path. Users are not

Table 2: User-cost definitions under different objective $m$.

| **Objective** $m$ | **User edge cost** $c^k_{m,e}$ | **Boarding cost** $c^{k,\text{in}}_{m,l,v,t}$ | **Alighting cost** $c^{k,\text{out}}_{m,l,v,t}$ |
|---|---|---|---|
| $m_{\text{vtt}}$ | 0 | 0 | 0 |
| $m_{\text{ivt}}$ | $\begin{cases} t'-t, & e\in\mathcal{E}^{\text{trv}} \\ 0, & e\in\mathcal{E}^{\text{hld}}\cup\mathcal{E}^{\text{tsf}} \\ +\infty, & e\in\mathcal{E}^{\text{src}}\cup\mathcal{E}^{\text{snk}} \end{cases}$ | 0 for $(l,v,t)\in\mathcal{N}^{\text{in}}$; $+\infty$ otherwise | 0 for $(l,v,t)\in\mathcal{N}^{\text{in}}$; $+\infty$ otherwise |
| $m_{\text{wait}}$ | $\begin{cases} t'-t, & e\in\mathcal{E}^{\text{hld}} \\ 0, & e\in\mathcal{E}^{\text{trv}}\cup\mathcal{E}^{\text{tsf}} \\ +\infty, & e\in\mathcal{E}^{\text{src}}\cup\mathcal{E}^{\text{snk}} \end{cases}$ | $\begin{cases} t-\text{IDT}^k-tt^w_{o^k,v}, & \text{if IDT}^k+tt^w_{o^k,v}<t, \\ +\infty, & \text{otherwise.} \end{cases}$ | 0 for $(l,v,t)\in\mathcal{N}^{\text{in}}$; $+\infty$ otherwise |
| $m_{\text{walk}}$ | 0 for $e\in\mathcal{E}^{\text{hld}}\cup\mathcal{E}^{\text{trv}}\cup\mathcal{E}^{\text{tsf}}$; $+\infty$ for $e\in\mathcal{E}^{\text{src}}\cup\mathcal{E}^{\text{snk}}$ | $\begin{cases} tt^w_{o^k,v}, & \text{if IDT}^k+tt^w_{o^k,v}<t, \\ +\infty, & \text{otherwise.} \end{cases}$ | $\begin{cases} tt^w_{v,d^k}, & (l,v,t)\in\mathcal{N}^{\text{in}} \\ +\infty, & \text{otherwise.} \end{cases}$ |
| $m_{\text{tsf}}$ | $\begin{cases} t'-t, & e\in\mathcal{E}^{\text{tsf}} \\ 0, & e\in\mathcal{E}^{\text{hld}}\cup\mathcal{E}^{\text{trv}} \\ +\infty, & e\in\mathcal{E}^{\text{src}}\cup\mathcal{E}^{\text{snk}} \end{cases}$ | 0 for $(l,v,t)\in\mathcal{N}^{\text{in}}$; $+\infty$ otherwise | 0 for $(l,v,t)\in\mathcal{N}^{\text{in}}$; $+\infty$ otherwise |
| $m_{\text{com}}$ | $\begin{cases} t'-t, & e\in\mathcal{E}^{\text{hld}}\cup\mathcal{E}^{\text{trv}}\cup\mathcal{E}^{\text{tsf}} \\ +\infty, & e\in\mathcal{E}^{\text{src}}\cup\mathcal{E}^{\text{snk}} \end{cases}$ | $\begin{cases} t-\text{IDT}^k & (l,v,t)\in\mathcal{N}^{\text{in}} \\ +\infty, & \text{otherwise.} \end{cases}$ | $\begin{cases} tt^w_{v,d^k}, & (l,v,t)\in\mathcal{N}^{\text{in}} \\ +\infty, & \text{otherwise.} \end{cases}$ |

obliged to follow that path; they might also find better paths in the DRT network we construct. The guaranteed path only ensures that there is at least a way for users to reach their destination while satisfying their requirements.

We introduce the following decision variables for our problem:

- $z_e\in\{0,1\}$: edge-activation indicator ($z_e=1$ iff edge $e$ is active).
- $x^k_e\in\{0,1\}$: user-edge usage indicator ($x^k_e=1$ iff user $k$ uses edge $e$).
- $b^k_{l,v,t}, h^k_{l,v,t}\in\{0,1\}$: boarding and alighting indicators for user $k$ at stop-time $(l,v,t)$ ($b^k_{l,v,t}=1$ iff user $k$ boards at $(l,v,t)$; 0 otherwise).

For implementation convenience, we also use:

- $a^k_{l,v,t} := b^k_{l,v,t}-h^k_{l,v,t}\in\{-1,0,1\}$: net flow indicator for users ($a^k_{l,v,t}=1$ if user $k$ boards at $(l,v,t)$; $-1$ if the user alights; 0 otherwise).
- $fw^k\in\{0,1\}$: fully-walking indicator; one may encode $fw^k=1-\sum_{(l,v,t)\in\mathcal{N}} b^k_{l,v,t}$ ($fw^k=1$ iff user $k$ never boards a DRT vehicle).

We formulate our problem as follows:

$$\min \quad \sum_{e\in\mathcal{E}} F_e z_e + \sum_{m\in\mathcal{M}} \left(\alpha_m \sum_{k\in\mathcal{K}} c^k_m\right). \tag{2}$$

$$\textstyle\sum_v z_{(src,(l,v,0))} = \sum_t\sum_v z_{s((l,v,t),snk)} = 1,\ \forall l\in\mathcal{L} \tag{3}$$

$$\textstyle\sum_{e\in\mathcal{E}^+_l(n)} z_e = \sum_{e\in\mathcal{E}^-_l(n)} z_e \le 1, \forall l\in\mathcal{L},\ n\in\mathcal{N}^{\text{in}}_l \tag{4}$$

$$\textstyle\sum_{e\in\mathcal{E}_l(n)} z_e + \sum_{e'\in\mathcal{E}_{l'}(n')} z_{e'} - 1 \ge z_e, \quad \forall e\in\mathcal{E}^{\text{tsf}} \tag{5}$$

$$\textstyle\sum_k x^k_e \ge z_e, \quad \forall e\in\mathcal{E}^{\text{tsf}} \tag{6}$$

$$x^k_e \le z_e, \quad \forall e\in\mathcal{E} \tag{7}$$

$$x^k_e \le 1-fw^k, \quad \forall k\in\mathcal{K},\ e\in\mathcal{E} \tag{8}$$

$$\textstyle\sum_n b^k_n = \sum_n h^k_n = 1-fw^k, \quad \forall k\in\mathcal{K} \tag{9}$$

$$\textstyle\sum_l\sum_t b^k_n + \sum_l\sum_t h^k_n <= 1, \quad \forall k\in\mathcal{K},\ v\in\mathcal{V} \tag{10}$$

$$fw^k = 0, \quad if\, tt^w_{o^k,d^k} > \text{FW}_{\text{MAX}}, \quad \forall k\in\mathcal{K} \tag{11}$$

$$\textstyle\sum_{e\in\mathcal{E}^+(n)} x^k_e - \sum_{e\in\mathcal{E}^-(n)} x^k_e = a^k_n \quad \forall k\in\mathcal{K} \tag{12}$$

$$\textstyle\sum_n b^k_n\cdot t \le \text{LPUT}^k, \quad \forall k\in\mathcal{K} \tag{13}$$

$$\textstyle\sum_n h^k_n\cdot(t+tt^w_{v,d^k}) \le \text{LAT}^k, \quad \forall k\in\mathcal{K} \tag{14}$$

The optimization function (2) minimizes the operator cost and user cost. Specifically, $F_e$ denotes the operator cost associated with edge $e$, and $c^k_m$ is the user cost for request $k$ under objective $m$ (see §3.1). The coefficients $\alpha_m$, with $\alpha_m\ge 0$, weight the relative importance of each component.

Constraints (3)-(5) are *DRT* constraints to ensure the graph of DRT is feasible. Constraints (3) ensure that each vehicle, aka each line $l$, starts a route from the super-source node *src* and ends at the super-sink node *snk*. Constraints (4) are line continuity equations to ensure that the route on a line is continuous. Constraints (5) ensure each node on a transfer edge is active. Constraint (6) adds an additional constraint for transfer edges that ensures transfer edges can only be active if any user uses it.

Constraints (7)-(14) are *user* constraints to ensure each user uses a feasible DRT service. Constraints (7) ensure users can only use active edges. Constraints (8)-(9) ensure users can only select edges and board and alight the DRT once if they do not walk by foot. Constraints (10) ensure each user cannot enter and exit the DRT system at the same physical stop $v$. Constraints (11) ensure that we consider walking from origin to destination feasible, only if the walking distance does not exceed the threshold $\mathrm{FW}_{\max}$. Otherwise, the model ensures that a path within DRT systems is constructed. Constraints (12) are the flow constraints for each user. Constraints (13) ensure that the pickup time of user $k$ at ingress time-stop $(l,v,t)$ is no later than their latest pickup time. Constraints (14) ensure that the dropoff time of user $k$ at egress time-stop $(l,v,t)$ plus their walking time to their destination is no later than their latest arrival time.

# 4 NUMERICAL RESULTS

We design two series of experiments to verify the feasibility of our approach. First, we conduct a small-scale experiment to compare our formulation against the PDPTW benchmark; second, we study the effect of different components of our formulation by component analysis.

## 4.1 Experimental Setting

All instances for the DRT problem were solved with the branch-and-cut mixed-integer linear programming (MILP) solver from `Gurobi 12.0.1` (linux64; AlmaLinux 9.6). Computations ran on a node with an Intel Xeon Gold 6226R @ 2.90,GHz (32 physical cores / 32 logical processors). We enabled up to 32 solver threads and imposed a per-instance time limit of 30 minutes. A representative instance contained up to 2,287,527 constraints (rows) and 1,269,755 variables (columns) and solved in about 12 minutes.

## 4.2 Evaluation Scenarios

We use the New York yellow taxi trip dataset from 2015 (New York City Taxi and Limousine Commission, 2015). From this dataset, we randomly sample several subsets of trips. Each instance of the problem thus takes as input a subset $\mathcal{K}$.

Given each $\mathcal{K}$, we first compute vehicle routes solving a VRP only specifying pickup and dropoff locations, without indication about the time. We use the Google OR-Tools solver (Google OR-Tools, 2025b) to obtain these optimal routes. From these routes, we derive each traveler's IDT, IAT, and corresponding LPUT and LAT. These derived values are then used to define the time window (TW) constraints for subsequent experiments. Note that, by building the user trips (input of the problem) based on the results from VRP, we are ensuring a conservative evaluation of our approach, in favor of VRP. Note also that (Google OR-Tools, 2025b) is used only to build the input to the problem (i.e., the set of trips), as described above, and will not be used as a benchmark, which is instead explained in the next paragraph.

Next, we compute a solution of the PDPTW (Dumas et al., 1991), which is a variant of VRP with explicit pickup and dropoff TW constraints, and we adopt this solution as our benchmark. We compute it via the correspond Google OR-Tools solver (Google OR-Tools, 2025a). This solver provides high-quality or near-optimal solutions and is widely used in MoD research (Kim et al., 2023). Since the scalability of our approach is left for future work, we deliberately chose this benchmark instead of heuristic or meta-heuristic algorithms to emphasize solution quality and use it as a strong benchmark to evaluate the performance of our methodology.

Observe that: (i) While the optimization function of PDPTW is the VTT, our optimization function (2) also includes user-related cost terms, without which we obtain solutions that are extremely efficient for the operator but expensive for users. However, we will show that our approaches outperform PDPTW's solutions also in terms of their objective, i.e., VTT; (ii) the TW constraints of our problem formulation are stricter than those in PDPTW. Specifically, LPUT bounds the sum of the user's first-mile walking time to the boarding station and user waiting time at the station until the bus arrives (Constraints 13); While in PDPTW, LPUT bounds only the waiting time at the pickup place. Similarly, in our formulation, LAT bounds the alighting time at the station plus the traveler's last-mile walking time (Constraints 14); While in PDPTW, LAT only bounds users' dropoff time. Despite our more restrictive constraints, our approaches outperform PDPTW due to designing a structured temporal network. It effectively consolidates demand, instead of calculating individual vehicle routes as in PDPTW.

A step-by-step overview of our experimental pipeline, along with a comparison between our methodology and the VRP benchmarks, is presented in Table 3.

Table 3: Pipeline for VRP benchmarking and proposed solution.

| Step | Method | Input | Output | Purpose |
|---|---|---|---|---|
| 1 | Google OR-Tools solver for classic VRP | Problem instance (vehicles, requests, distances), without TW constraints | IDT, IAT, LPUT, and LAT of each user | Establish baseline feasibility ignoring TW constraints; generate IDT, IAT, LPUT, and LAT (aka TW constraints) for sequential experiments |
| 2 | Google OR-Tools solver for PDPTW | Same problem instance but with TW constraints | Feasible VRP solution | Generate benchmark solution with TW constraints to compare against our method |
| 3 | Proposed method | Same problem instance with TW constraints | Solution generated | Demonstrate performance relative to the PDPTW benchmark |

## 4.3 Performance Indicators

Baier et al. (2024) provides a comprehensive review of measuring the performance of DRT systems from different dimensions (environmental, service, political, and financial dimensions) and different stakeholders' viewpoints.

In our work, the passenger-related measurements focus on connectivity and directness, including user in-vehicle traveled time (IVT), waiting, transfer time, and walking time to board and alight stops or fully walk.

In terms of supply measurements, we mainly use VKT, which is a proxy of the operating cost (Tirachini and Gomez-Lobo, 2020).

To evaluate system performance, we consider three key performance indicators (KPIs): system efficiency (SE), vehicle utilization (VU), and user arrival time delay (AD).

**SE** (Liebchen et al., 2021) measures the total reduction in travel distance achieved by the DRT system compared to direct private car trips. It is defined as $\mathrm{SE} = \mathrm{VKT}_{\mathrm{veh}}/\mathrm{VKT}_{\mathrm{direct}}$, where $\mathrm{VKT}_{\mathrm{direct}}$ is the total VKT for a hypothetical direct private-car trip, and $\mathrm{VKT}_{\mathrm{veh}}$ is the total VKT operated by the DRT fleet. If the SE value is less than 1, it indicates that the DRT system reduces the overall travel distance compared to the use of a private car.

**VU** (Barth, 2025) measures the average onboard occupancy. It is defined as $\mathrm{VU} = \mathrm{VKT}_{\text{in-veh}}/\mathrm{VKT}_{\mathrm{veh}}$, where $\mathrm{VKT}_{\text{in-veh}}$ is the total in-vehicle kilometers traveled by users. A higher VU value indicates higher vehicle occupancy and utilization efficiency.

**AD** measures how much delay user experience in the DRT system compared with the hypothetical direct private-car trip. It is defined as $\mathrm{AD}^k = \mathrm{AAT}^k - \mathrm{IAT}^k$, where $\mathrm{AAT}^k$ is user $k$'s actual arrival time.

## 4.4 Benchmark Comparison

First, we compare the performance of our approach against the PDPTW benchmark. As shown in Figure 1, our method under different objective $m$ consistently reduces vehicle VKT and user IVT relative to the benchmark (denoted as $\mathrm{GOR}_2$ in the plot), while maintaining reasonable waiting, transfer, and walking times across different objectives. Moreover, our approach demonstrates a significant increase in SE compared to the benchmark, indicating that our methodology achieves greater total travel distance savings. However, these improvements come with trade-offs in VU and AD, as our approach generally exhibits higher delays and negative VU gains relative to the benchmark.

In summary, our methods show higher reduction in detours and higher efficiency in VKT, but also have higher delay in travel time and lower occupancy rate.

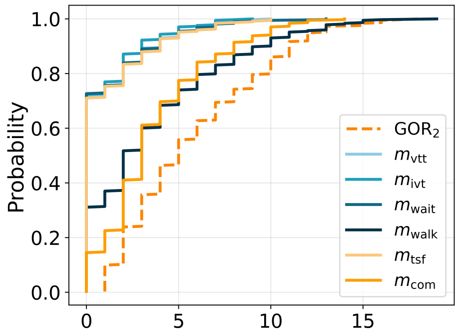
## 4.5 Component Analysis

### 4.5.1 Impact of Walking

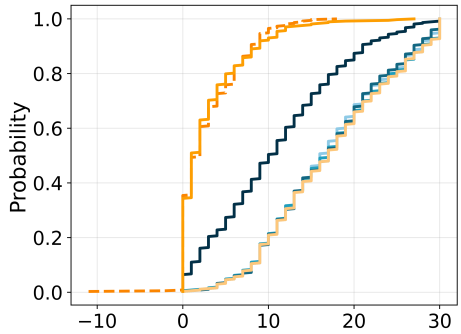
According to Figure 2, after removing walking, both VKT and IKT increases significantly across all objectives. Regarding user IVT, the objective $m_{\mathrm{ivt}}$ and objective $m_{\mathrm{com}}$ still maintain lower IVT values than the benchmark. For VKT, all objectives rise to a level comparable to the benchmark. This indicates that walking within a reasonable distance helps reduce detours for users as well as vehicles.

In terms of AD, providing door-to-door services through the PT system slightly reduces user delays for most objectives. An exception occurs with the composite objective ($m_{\mathrm{com}}$), where AD increases marginally but remains below the benchmark, regardless of whether walking is considered. This outcome may be related to the absence of an explicit penalty for arrival delay in the cost function.

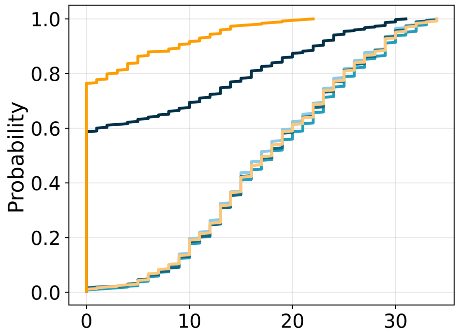
Notably, for $m_{\mathrm{walk}}$ and $m_{\mathrm{com}}$, before removing



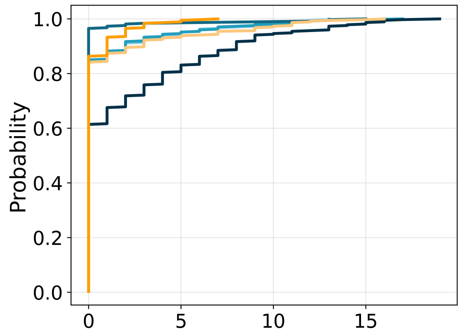

(a) User IVT



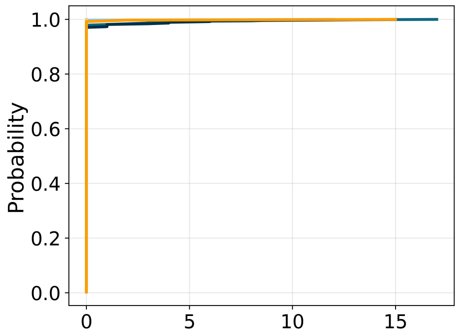

(b) AD (min)



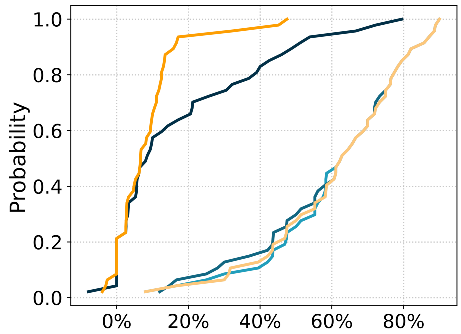

(c) Walking time (min)



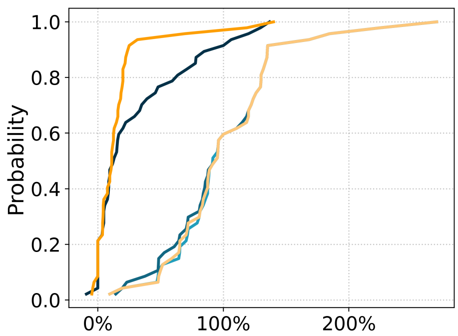

(d) Waiting time (min)



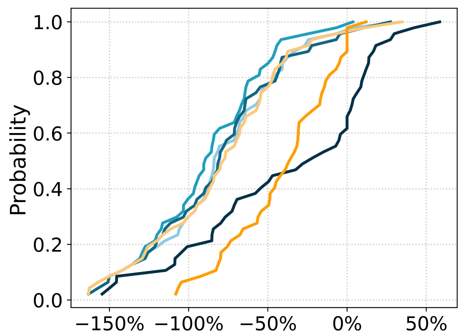

(e) Transfer time (min)




(f) Percentage reductions in VKT (vs. PDPTW)




(g) Relative SE gain (vs. PDPTW)

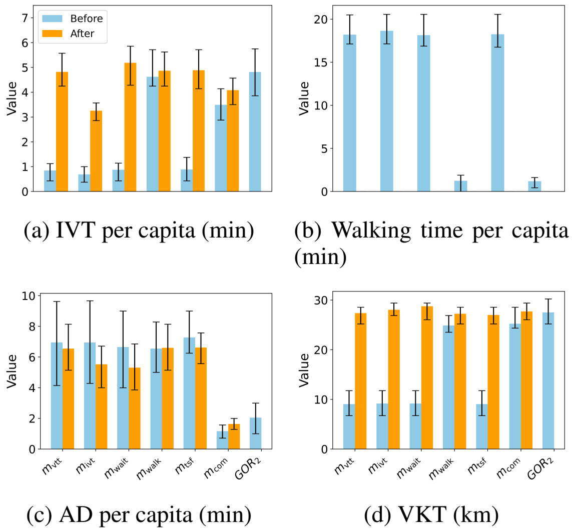



(h) Relative VU gain (vs. PDPTW)

Figure 1: Cumulative density distribution (CDF) of different objectives.

walking, the walking distance was only indirectly constrained via its penalty term in the optimization function. Thus, users could still travel entirely on foot when their walking cost was lower than the operator cost.

In summary, enabling walking introduces an acceptable increase in walking time and delay for users, while substantially decreasing VKT and IVT.




(a) IVT per capita (min)

(b) Walking time per capita (min)

(c) AD per capita (min)

(d) VKT (km)

Figure 2: Component analysis under the no-walking scenario. Error bars represent the interquartile range (25th–75th percentile).

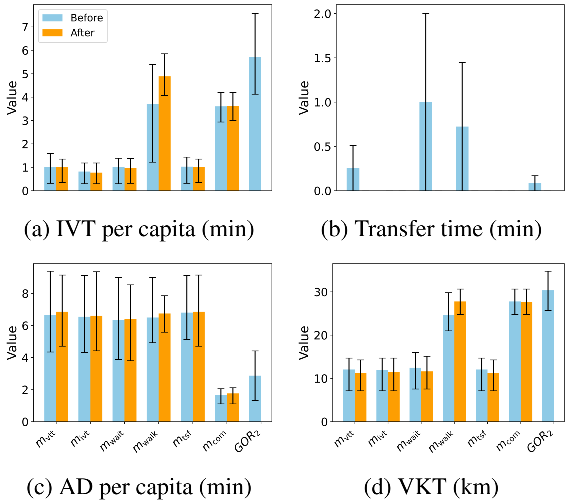
#### 4.5.2 Impact of Transfer

Table 4: Percentage of trips by number of transfers for each objective.

| Objective $m$ | 0 transfer (%) | 1 transfer(%) | 2 transfers (%) | >2 transfers (%) |
|---|---|---|---|---|
| $m_{vtt}$ | 99.73 | 0.00 | 0.27 | 0.00 |
| $m_{ivt}$ | 100.00 | 0.00 | 0.00 | 0.00 |
| $m_{wait}$ | 97.86 | 0.54 | 1.07 | 0.54 |
| $m_{walk}$ | 97.05 | 2.95 | 0.00 | 0.00 |
| $m_{tsf}$ | 100.00 | 0.00 | 0.00 | 0.00 |
| $m_{com}$ | 98.93 | 1.07 | 0.00 | 0.00 |

According to Figure 3, there is no transfers after imposing the transfer constraints. However, even before that, most cases do not involve transfers (Table 4), except for objectives $m_{\text{walk}}$. This indicates that the current trip sample may not be sufficiently diverse to capture the impact of transfer. For objective $m_{\text{walk}}$, we clearly observe that, without transferring, IVT and VKT increase significantly. While AD remains relatively stable in all objectives.

In summary, the component analysis of transfers did not yield insightful findings given the small scale of the experiments and lack of transfer options. A larger experimental scale is necessary to capture more diverse transfer patterns. This will be explored in future research.




(a) IVT per capita (min)

(b) Transfer time (min)

(c) AD per capita (min)

(d) VKT (km)

Figure 3: Component analysis under the no-transfer scenario. Error bars represent the interquartile range (25th–75th percentile).

## 5 CONCLUSIONS

This work proposes a novel formulation for the DRT optimization problem from a temporal network design perspective. Unlike VRP, our approach constructs a time-expanded transit network that explicitly integrates routes, timetables, walking, and transfers. A proof-of-concept implementation based on an ILP formulation was developed to verify the feasibility of the proposed framework. A series of small-scale experiments using real-world taxi trip data was conducted to benchmark our approach against classical VRP solutions. The results show that our proposed method can significantly reduce both vehicle- and in-vehicle kilometers traveled, thereby improving system efficiency while maintaining reasonable levels of arrival delay and vehicle utilization. Component analysis further highlights the benefits of incorporating walking into the DRT system, as both contribute to reductions in VKT and user IVT. While for the component analysis of transferring, it requires larger-scale or more diverse samples to capture more transferring behaviors.

Overall, the numerical results indicate that temporal network design provides a promising alternative framework for optimizing flexible transit systems. By shifting the focus from vehicle tours to time-dependent network structures, DRT services can achieve a more balanced performance between operation efficiency and user experience.

Future research can focus on scalability, dynamism, and demand modeling.

This work serves primarily as a proof-of-concept; large-scale experiments to assess computational scalability and solution robustness are left for future work. Methods such as clustering could be explored to enable large network applications.

In addition, this work mainly addresses offline planning for DRT, where user demand is assumed to be known in advance. In scenarios with uncertain or unknown future demand, the offline planning framework would need to be extended to an online dynamic setting, for which approaches based on reinforcement learning could be appropriate.

Finally, but not least, the integration of discrete choice modeling is also a promising direction to capture more realistic user decision-making processes, allowing users to select transportation modes and accept or reject system assignments based on their individual preferences.

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